Dynamics of filament length

$$\frac{dn}{dt} = k_{on} \cdot c_{AM} - k_{off}$$

$c_{AM} :=$ concentration of available monomer

$$c_{AM} = c_0 - \frac{M}{V} \underline{n(t)}$$

$$\frac{dn}{dt} = k_{on} \cdot \left(c_0 - \frac{M}{V} n(t)\right) - k_{off}$$

$$= \underbrace{k_{on} \cdot c_0 - k_{off}}_{\alpha} - \underbrace{\frac{M}{V} \cdot k_{on}}_{\beta} \cdot n(t)$$

$$\frac{dn}{dt} = \alpha - \beta n(t)$$

Ansatz: $e^{-\kappa t} \cdot \frac{d}{dt}\left(e^{\kappa t} \cdot f(t)\right) = \underline{\frac{d}{dt} f(t) + \kappa \cdot f(t)}$

$\kappa = \beta \quad ; \quad f(t) = n(t)$

$$\frac{dn}{dt} + \beta n(t) = \alpha$$

$$\cancel{e^{-\beta t}} \cdot \frac{d}{dt}\left(e^{\beta t} n(t)\right) = \alpha e^{\beta t} \qquad \int_0^t$$

$n(0) = 0$

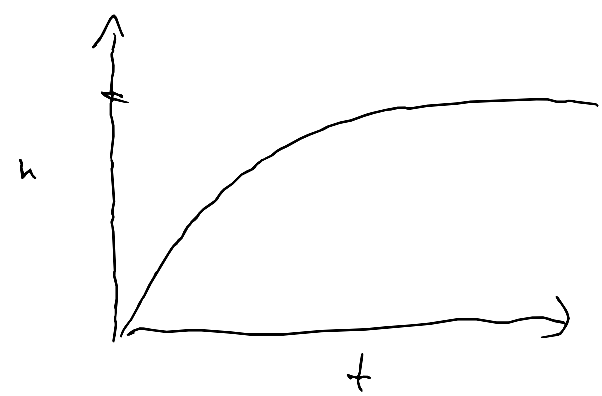
$$e^{\beta t} \cdot n(t) - \cancel{n(0)} = \frac{\alpha}{\beta} \cdot \left(e^{\beta t} - 1\right)$$

$$n(t) = \frac{\alpha}{\beta} \cdot \left(1 - e^{-\beta t}\right)$$

Recall $\alpha = k_{on} \cdot c_0 - k_{off}$

$\beta = k_{on} \cdot \frac{M}{V}$

$$n(t) = \frac{k_{on} \cdot c_0 - k_{off}}{k_{on} \cdot M} \cdot V \cdot \left(1 - e^{-k_{on} \cdot \frac{M}{V} \cdot t}\right)$$



$$n\left(t \gg \frac{V}{M k_{on}}\right) = \frac{k_{on} \cdot c_0 - k_{off}}{k_{on} \cdot M} \cdot V$$

$$n(t \text{ small}) = (k_{on} \cdot c_0 - k_{off})\, t$$