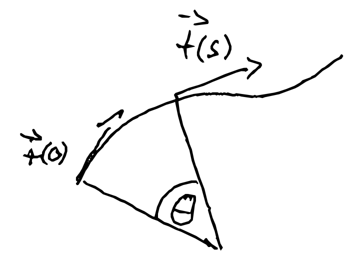
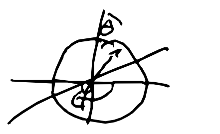
Persistence Length



$\underset{\sim\sim\sim\sim\sim}{\langle \vec{t}(0)\, \vec{t}(s) \rangle} = ?$

$\vec{t}(0) \cdot \vec{t}(s) = \cos\theta$ for small θ => $\cos\theta = 1 - \frac{\theta^2}{2}$

$\langle \vec{t}(0) \vec{t}(s) \rangle = 1 - \frac{\langle\theta^2\rangle}{2}$



We search variance of θ

$$\langle\theta^2\rangle = \frac{\int \theta^2 \cdot p(\theta)\, d\Omega}{\int p(\theta)\, \Omega}$$

$d\Omega :=$ solid angle $\sin\theta \cdot d\theta \cdot d\varphi$

$\beta = \frac{1}{k_B T}$

$E_b = \frac{1}{2} \kappa_f \cdot \frac{1}{R^2} \cdot L_c$

$L_c = R \cdot \theta \Rightarrow R = \frac{L_c}{\theta}$

$E_b = \frac{1}{2} \cdot \kappa_f \cdot \frac{\theta^2}{L_c}$

Use Boltzmann $p(E(\theta)) = p_0 \cdot \exp(-\beta \cdot E(\theta))$

$$\Rightarrow \langle\theta^2\rangle = \frac{\int_0^{2\pi}\int_0^{\pi} \theta^2 \cdot \cancel{p_0} \cdot \exp\left(-\beta \cdot \frac{\kappa_f \cdot \theta^2}{2 L_c}\right) \cdot \cancel{\sin\theta}^{\,\theta} \cdot d\theta \cdot \cancel{d\varphi}}{\int_0^{2\pi}\int_0^{\pi} \cancel{p_0} \cdot \exp\left(-\beta \frac{\kappa_f \theta^2}{2 \cdot L_c}\right) \cdot \cancel{\sin\theta}_{\,\theta} \cdot d\theta \cdot \cancel{d\varphi}}$$

$$= \frac{\int_0^{\pi} \theta^3 \cdot \exp\left(-\beta \frac{\kappa_f}{2L_c} \cdot \theta^2\right) d\theta}{\int_0^{\pi} \theta \cdot \exp\left(-\beta \frac{\kappa_f}{2L_c} \cdot \theta^2\right) d\theta}$$

$x = \sqrt{\beta \frac{\kappa_f}{2L_c}} \cdot \theta$

$d\theta = \sqrt{\frac{2L_c}{\kappa_f \beta}} \cdot dx$

$$= \frac{2L_c}{\beta \kappa_f} \cdot \frac{\int_0^{\infty} x^3 \cdot \exp(-x^2)\, dx}{\int_0^{\infty} x \cdot \exp(-x^2)\, dx}$$

$\int_0^{\infty} x^3 \cdot \exp(-x^2)\, dx = \frac{1}{2}$

$\int_0^{\infty} x \cdot \exp(-x^2)\, dx = \frac{1}{2}$

=> Look at $L_c = s$

$$\langle\theta^2\rangle = \frac{2s}{\beta \kappa_f}$$

$$\langle \vec{t}(0) \vec{t}(s) \rangle = 1 - \frac{\langle\theta^2\rangle}{2} = 1 - \frac{s}{\beta \kappa_f}$$

s is small compared to $\beta \cdot \kappa_f = L_p$

$$\boxed{\langle \vec{t}(0) \vec{t}(s) \rangle \approx \exp\left(-\frac{|s|}{L_p}\right)}$$

$\exp\left(-\frac{s}{L_p}\right) = 1 - \frac{s}{L_p}$

Using $\boxed{L_p = \beta \cdot \kappa_f = \frac{\kappa_f}{k_B T}}$

Search $p(x)$ probability for certain end-to-end distance x

N total steps, r to right and l steps to left

$\underline{n}$: = final position in units of a $\quad (x = n \cdot a)$

$N = r + l \Rightarrow l = N - r$

$n = r - l \qquad r = N - l$

$n = r - N + r \Rightarrow \boxed{r = \frac{N+n}{2}}$

$n = N - l - l \Rightarrow \boxed{l = \frac{N-n}{2}}$

Need to distribute r, l „Balls" at N bins

$$p(n) = \frac{N!}{\left(\frac{N+n}{2}\right)! \cdot \left(\frac{N-n}{2}\right)!}$$

Sterling $\ln(x!) \approx x \cdot \ln x - x$

$$\ln(p(n)) = N \cdot \ln(N) - \cancel{N} - \frac{N+n}{2} \ln\left(\frac{N+n}{2}\right) + \cancel{\frac{N+n}{2}} - \frac{N-n}{2} \ln\left(\frac{N-n}{2}\right) + \cancel{\frac{N-n}{2}}$$

$\boxed{\text{Taylor } (x \pm \varepsilon) \cdot \ln(x \pm \varepsilon) = x \ln(x) \pm \varepsilon \cdot (\ln(x) + 1) + \frac{1}{2}\varepsilon^2 \frac{1}{x}}$

$$\ln(p(n)) = N \cdot \ln(N) - \frac{N}{\cancel{2}} \cdot \ln\left(\frac{N}{2}\right) - \frac{n}{2}\left(\cancel{\ln\left(\tfrac{N}{2}\right) + 1}\right) - \frac{1}{2} \cdot \frac{n^2}{4} \cdot \frac{2}{N}$$

$$- \frac{N}{2} \cdot \cancel{\ln\left(\tfrac{N}{2}\right)} + \frac{n}{2}\left(\cancel{\ln\left(\tfrac{N}{2}\right) + 1}\right) - \frac{1}{2} \frac{n^2}{4} \frac{2}{N}$$

$$= N \cdot \cancel{\ln(N)} - N \cdot \cancel{\ln(N)} + N \ln(2) - \frac{n^2}{2N} \quad \Big| \exp$$

$$\Rightarrow p(n) = \underbrace{\exp(N \ln(2))}_{p_0} \cdot \exp\left(-\frac{n^2}{2N}\right)$$

$x = n \cdot a \Leftrightarrow n = \frac{x}{a}$

$$p(x) \propto \exp\left(-\frac{x^2}{\underbrace{2Na^2}_{\sigma^2}}\right) \qquad \sigma = Na^2$$