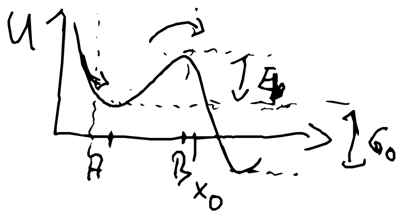
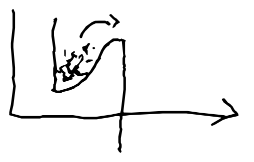


$j(A) = j(B) = j_0$ $\qquad \gamma v = F_x = -\frac{dU}{dx}$

$c(x_0) = 0$



$$j(x) = v(x)\cdot c(x) - D\frac{dc}{dx} = j_0 \qquad D = \frac{k_BT}{\gamma}$$

$$j_0 = -\frac{c(x)}{\gamma}\cdot\frac{dU}{dx} - D\frac{dc}{dx} \quad |\cdot\gamma$$

$$\boxed{-\gamma j_0 = c(x)\frac{dU}{dx} + \gamma D\frac{dc}{dx}}$$

$$\phi(x) = c(x)\cdot\exp\left(\frac{U(x)}{k_BT}\right)$$

Derivative $\frac{d\phi}{dx} = c(x)\cdot\frac{1}{k_BT}\cdot\frac{dU}{dx}\exp\left(\frac{U(x)}{k_BT}\right) + \exp\left(\frac{U(x)}{k_BT}\right)\cdot\frac{dc}{dx}$

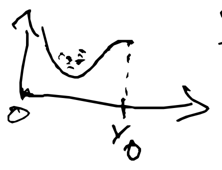
$$\Rightarrow \frac{d\phi}{dx} = \frac{1}{k_BT}\cdot\exp\left(\frac{U(x)}{k_BT}\right)\left(c(x)\cdot\frac{dU}{dx} + D\gamma\frac{dc}{dx}\right)$$

$$k_BT\exp\left(-\frac{U(x)}{k_BT}\right)\cdot\frac{d\phi}{dx} = c(x)\frac{dU}{dx} + D\gamma\frac{dc}{dx}$$

$$\Rightarrow -\frac{j_0}{D}\cdot\exp\left(\frac{U(x)}{k_BT}\right) = \frac{d}{dx}\left(c(x)\cdot\exp\left(\frac{U(x)}{k_BT}\right)\right)$$

Eq. $j_0 = 0 \Rightarrow \quad 0 = \frac{d}{dx}\left(c(x)\cdot\exp\left(\frac{U(x)}{k_BT}\right)\right) \Rightarrow c(x) = c_0\cdot\exp\left(-\frac{U(x)}{k_BT}\right)$

$\hookrightarrow$ Boltzmann



Perfect absorber $\Rightarrow$ $c(x_0) = 0$

$$\int_x^{x_0} -\frac{j_0}{D}\cdot\exp\left(\frac{U(x')}{k_BT}\right)dx' = c(x_0)\,\cancel{\exp\left(\frac{U(x_0)}{k_BT}\right)} - c(x)\cdot\exp\left(\frac{U(x)}{k_BT}\right)$$

$$\Rightarrow \boxed{c(x) = \frac{j_0}{D}\exp\left(-\frac{U(x)}{k_BT}\right)\cdot\int_x^{x_0}\exp\left(\frac{U(x')}{k_BT}\right)dx'}$$

$c_T = \int_0^{x_0} c(x)\,dx \qquad \Leftrightarrow \quad j_0 = k_{on}\,c_T$

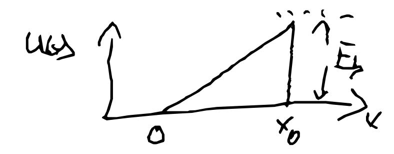
$$c_T = \int_0^{x_0}\frac{j_0}{D}\cdot\exp\left(-\frac{U(x)}{k_BT}\right)\cdot\int_x^{x_0}\exp\left(\frac{U(x')}{k_BT}\right)dx'\,dx$$

$$k_{on} = \frac{D}{\int_0^{x_0}\exp\left(-\frac{U(x)}{k_BT}\right)\cdot\int_x^{x_0}\exp\left(\frac{U(x')}{k_BT}\right)dx'\,dx}$$

Simple case $\quad U(x) = 0$

$$k_{on} = \frac{D}{\frac{1}{2}x_0^2} = \frac{2D}{x_0^2} \qquad \text{Recall } k_{on} = \frac{1}{\tau_{on}}$$

$$\Rightarrow \boxed{x_0^2 = 2D\tau_{on}}$$



$U(x) = -x\cdot F \quad \Rightarrow \quad E_b = -x_0\cdot F$

$$\tau_{on} = \frac{1}{D}\int_0^{x_0}\exp\left(\frac{x\cdot F}{k_BT}\right)\int_x^{x_0}\exp\left(-\frac{x'F}{k_BT}\right)dx'\,dx$$

$$\tau_{on} = \frac{1}{D}\cdot\int_0^{x_0}\exp\left(\frac{xF}{k_BT}\right)\frac{-F}{k_BT}\left[\exp\left(\frac{E_b}{k_BT}\right) - \exp\left(-\frac{xF}{k_BT}\right)\right]dx$$

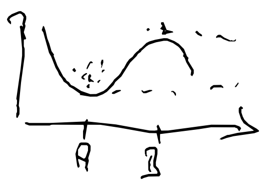
$$\boxed{\tau_{on} = \left(\frac{k_BT}{E_b}\right)^2\frac{x_0^2}{D}\left[-1 + \exp\left(\frac{E_b}{k_BT}\right) + \frac{E_b}{k_BT}\right]}$$

Limit $E_b \gg k_BT$

$$= \left(\frac{k_BT}{E_b}\right)^2\frac{x_0^2}{D}\cdot\exp\left(\frac{E_b}{k_BT}\right) \Leftrightarrow k_{on} = \left(\frac{E_b}{k_BT}\right)^2\cdot\frac{D}{x_0^2}\cdot\exp\left(-\frac{E_b}{k_BT}\right)$$

$\int\exp(-\alpha x^2)dx = \sqrt{\frac{\pi}{\alpha}}$

Kramers Rate in Literature



$U(A+\delta) = U(A) + \frac{1}{2}U''(A)\cdot\delta^2$ $\quad$ Expansion

$$\int_0^{x_0}\exp\left(-\frac{U(x)}{k_BT}\right)dx \approx \int_{-\infty}^{\infty}\exp\left(-\frac{U(A)}{k_BT}\right)\cdot\exp\left(-\frac{U''(A)\cdot\delta^2}{2k_BT}\right)d\delta$$

$$\int_x^{x_0}\exp\left(\frac{U(x)}{k_BT}\right)dx \approx \int_{-\infty}^{\infty}\exp\left(\frac{U(B)}{k_BT}\right)\cdot\exp\left(-\frac{|U''(B)|\cdot\delta^2}{2k_BT}\right)d\delta$$

$$k_{on} = \frac{D}{\int_0^{x_0}\exp\left(-\frac{U(x)}{k_BT}\right)\int_x^{x_0}\exp\left(\frac{U(x')}{k_BT}\right)dx'\,dx} \approx \frac{D}{\exp\left(\frac{\Delta U}{k_BT}\right)\cdot\sqrt{\frac{\pi}{U''(A)}\cdot 2k_BT}\cdot\sqrt{\frac{\pi}{|U''(B)|}\cdot 2k_BT}}$$

$$\boxed{k_{on,Kramers} = \frac{D}{2\pi k_BT}\cdot\sqrt{U''(A)\cdot|U''(B)|}\cdot\exp\left(-\frac{\Delta U}{k_BT}\right)}$$