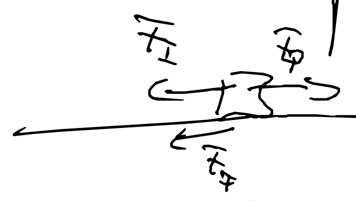
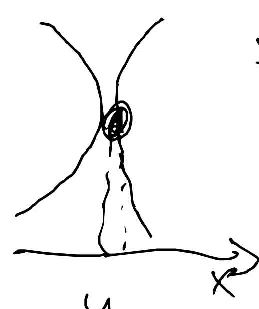
Langevin approach to ~~c~~ calibrate optical tweezer





$F_p = -k\cdot x$

$$m\cdot\frac{dv}{dt} = -\gamma\frac{dx}{dt} + F_p + F_{Th} = 0$$

$$\gamma\frac{dx}{dt} = -k\cdot x + \sqrt{2\gamma k_B T}\,\xi(t)$$

$$\dot{x} = -\frac{k}{\gamma}x + \sqrt{2D}\cdot\xi(t)$$

$F_{Th} = \sqrt{2\gamma k_B T}\cdot\xi(t)$

$\gamma :=$ Friction coeffict $= 6\pi\eta R$

$\langle\xi(t)\rangle = 0$

$\langle\xi(0)\xi(t)\rangle = \delta(t)$

Wiener Khinchin Theorem:

$\boxed{FT(R(t)) = PSD}$

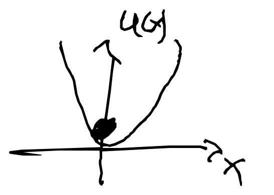
$\hookrightarrow FT$

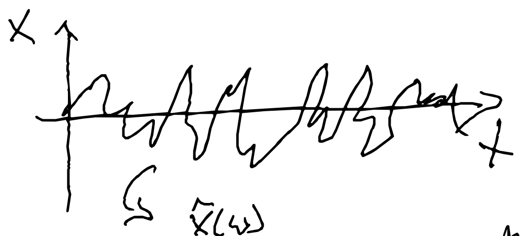
$$i\omega\,\tilde{x}(\omega) = -\frac{k}{\gamma}\tilde{x}(\omega) + \sqrt{2D}\,\tilde{\xi}(\omega)$$

$$\tilde{x}(\omega) = \frac{\sqrt{2D}}{i\omega + \frac{k}{\gamma}}\cdot\tilde{\xi}(\omega)$$

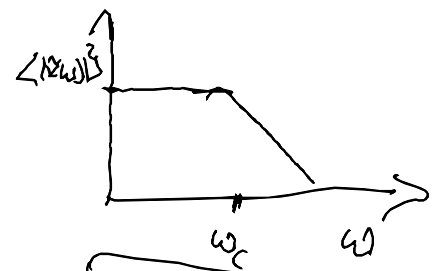
$$\langle|\tilde{x}(\omega)|^2\rangle = \frac{2D}{\left(i\omega+\frac{k}{\gamma}\right)\left(-i\omega+\frac{k}{\gamma}\right)}\cdot\langle|\tilde{\xi}(\omega)|^2\rangle$$

$$\langle|\tilde{x}(\omega)|^2\rangle = \frac{2D}{\left(\frac{k}{\gamma}\right)^2+\omega^2}\cdot 1$$



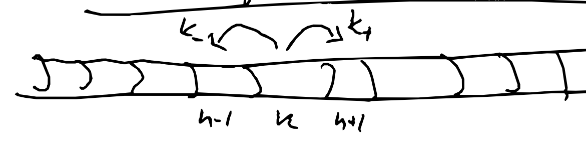


$\hookrightarrow \tilde{x}(\omega)$



$\boxed{\omega_c = \frac{k}{\gamma}}$

Master Equation for one state motor



$$p(n, t+\Delta t) = p(n-1,t)\cdot k_+\Delta t + p(n+1,t)k_-\Delta t + p(n,t)\cdot(1-k_+\Delta t - k_-\Delta t)$$

Step size of a $\quad n\cdot a = x$

$$p(x,t+\Delta t) = p(x-a,t)\,k_+\Delta t + p(x+a,t)k_-\Delta t + p(x,t)(1-k_+\Delta t-k_-\Delta t) \qquad \Big|-p(x,t)\Big|:\Delta t$$

Taylor:

$$p(x\pm a) = p(x) \pm a\frac{\partial p}{\partial x} + \frac{1}{2}a^2\frac{\partial^2 p}{\partial x^2}$$

$$\frac{p(x,t+\Delta t)-p(x,t)}{\Delta t} = \frac{\partial p}{\partial t} = \cancel{p(x)k_+} - a k_+\frac{\partial p}{\partial x} + \frac{1}{2}a^2 k_+\frac{\partial^2 p}{\partial x^2} + \cancel{p(x)k_-} + a k_-\frac{\partial p}{\partial x} + \frac{1}{2}a^2 k_-\frac{\partial^2 p}{\partial x^2} - \cancel{p(x)k_+} \cancel{p(x)k_-}$$

$$\Rightarrow \frac{\partial p}{\partial t} = a\cdot(k_- - k_+)\frac{\partial p}{\partial x} + \frac{1}{2}a^2(k_+ + k_-)\frac{\partial^2 p}{\partial x^2}$$

$$= -a(k_+ - k_-)\frac{\partial p}{\partial x} + \frac{1}{2}a^2(k_+ + k_-)\frac{\partial^2 p}{\partial x^2}$$

Compare to Smoluchowski

$$\frac{dc}{dt} = -v\frac{\partial c}{\partial x} + D\frac{\partial^2 c}{\partial x^2}$$

$$v = a(k_+ - k_-)$$

$$D = \frac{1}{2}a^2(k_+ + k_-)$$

To solve use change of ref. frame

$t' = t$

$x' = x - vt$

~~Tran~~ Transforming the derivatives

$$\frac{\partial p}{\partial t} = \frac{\partial p}{\partial t'}\frac{\partial t'}{\partial t} + \frac{\partial p}{\partial x'}\frac{\partial x'}{\partial t'} = \frac{\partial p}{\partial t'} - v\frac{\partial p}{\partial x'}$$

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial p}{\partial t'}\frac{\partial t'}{\partial x} = \frac{\partial p}{\partial x'}$$

$$\frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial x'^2}$$

$$\Rightarrow \frac{\partial p}{\partial t'} - v\frac{\partial p}{\partial x'} = -v\frac{\partial p}{\partial x'} + D\frac{\partial^2 p}{\partial x'^2}$$

Results in classical diffusion Eq.

$$\frac{\partial p}{\partial t'} = D\frac{\partial^2 p}{\partial x'^2} \qquad \text{with known solution}$$

moving frame: $\quad p(x',t) = \frac{1}{\sqrt{4\pi D t'}}\exp\left(-\frac{x'^2}{4Dt'}\right)$

rest frame $\quad p(x,t) = \frac{1}{\sqrt{4\pi D t}}\exp\left(-\frac{(x-vt)^2}{4Dt^2}\right)$