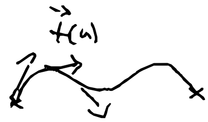
End to end distance of a wormlike chain



$$\boxed{\langle \vec{t}(u)\cdot\vec{t}(s)\rangle = \exp\left(-\frac{|s|}{L_p}\right)}$$

$$\langle r_{ee}^2\rangle = \left\langle \left(\int_0^{L_c} \vec{t}(u)\,du\right)^2 \right\rangle$$

$$= \left\langle \int_0^{L_c}\int_0^{L_c} \vec{t}(u)\cdot\vec{t}(v)\,dv\,du \right\rangle$$

$$= \int_0^{L_c}\int_0^{L_c} \langle \vec{t}(u)\cdot\vec{t}(v)\rangle\,dv\,du$$

$$= \int_0^{L_c}\int_0^{L_c} \exp\left(-\frac{|u-v|}{L_p}\right)dv\,du$$

$$= 2\int_0^{L_c}\int_0^{u} \exp\left(-\frac{u-v}{L_p}\right)dv\,du$$

$$= 2\int_0^{L_c} \exp\left(-\frac{u}{L_p}\right)\int_0^{u} \exp\left(\frac{v}{L_p}\right)dv\,du$$

$$= 2\int_0^{L_c} \exp\left(-\frac{u}{L_p}\right)\cdot L_p\cdot\left[\exp\left(\frac{u}{L_p}\right)-1\right]du$$

$$= 2L_p\int_0^{L_c}\left(1-\exp\left(-\frac{u}{L_p}\right)\right)du$$

$$= 2L_p\cdot\left[L_c-(-L_p)\cdot\left(\exp\left(-\frac{L_c}{L_p}\right)-1\right)\right]$$

$$\boxed{\langle r_{ee}^2\rangle = 2L_pL_c-2L_p^2\left(1-\exp\left(-\frac{L_c}{L_p}\right)\right)}$$