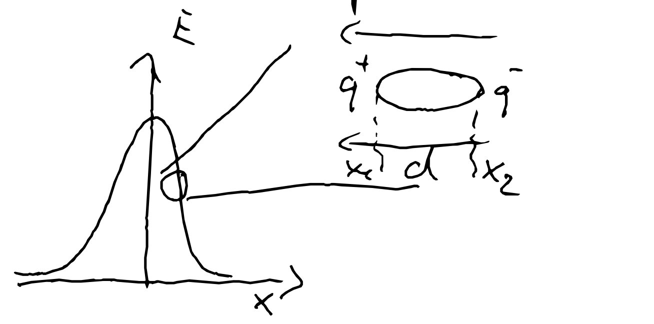
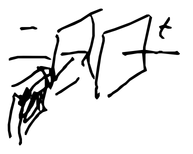
EM field based explanation to optical tweezers ?





$d = x_2 - x_1$

$\Rightarrow \boxed{\vec{p} = \alpha \cdot \vec{E} = q \cdot d}$

Force on a charge q in EM

$$\vec{F} = q\left(\vec{E} + \frac{dx}{dt} \times \vec{B}\right)$$

$$\vec{F}_T = q\left(\vec{E}(x_1) - \vec{E}(x_2) + \frac{d(x_1 - x_2)}{dt} \times \vec{B}\right)$$

Approximate $\vec{E}(x_2) = \vec{E}(x_1) + d \cdot \frac{d\vec{E}}{dx}\Big|_{x_1}$

$$= \vec{E}(x_1) + [(x_2 - x_1) \cdot \nabla]\vec{E}$$

$$\hookrightarrow \vec{F}_T = q\left(\cancel{\vec{E}(x_1)} - \cancel{\vec{E}(x_1)} + [(x_1 - x_2)\nabla]\vec{E} + \frac{d(x_1 - x_2)}{dt} \times \vec{B}\right)$$

$$= (p\nabla)\vec{E} + \frac{dp}{dt} \times \vec{B}$$

$$= \alpha \cdot \left[(\vec{E}\nabla)\vec{E} + \frac{d\vec{E}}{dt} \times \vec{B}\right]$$

Recall: $(\vec{E}\nabla)E = \nabla\left(\frac{1}{2}E^2\right) - E \times (\nabla \times E)$

Maxwell

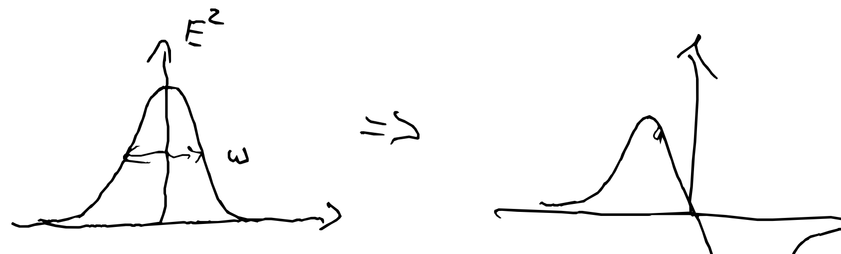
$$\frac{d\vec{B}}{dt} = -\nabla \times \vec{E}$$

$$\vec{F} = \alpha\left[\frac{1}{2}\nabla E^2 - E \times (\nabla \times E) + \frac{dE}{dt} \times \vec{B}\right]$$

$$= \alpha\left[\frac{1}{2}\nabla E^2 + \vec{E} \times \frac{d\vec{B}}{dt} + \frac{d\vec{E}}{dt} \times \vec{B}\right]$$

$$= \alpha\left[\frac{1}{2}\nabla E^2 + \cancel{\frac{d}{dt}\left(\vec{E} \times \vec{B}\right)}\right]$$

$$F = \frac{1}{2}\alpha \nabla E^2 = \frac{1}{2}\alpha \nabla I$$



$$I = I_0 \cdot e^{-\frac{x^2}{2w^2}}$$

$\hookrightarrow$ For small x ($x < w$)

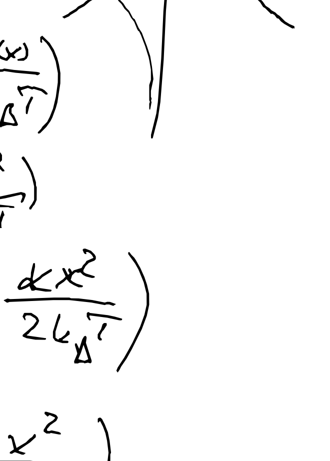
$$F = -\underbrace{\frac{I_0 \cdot \alpha}{2w^2}}_{\kappa} \cdot x$$

How to obtain κ

(1) Drag Force

$$F = \underline{\Delta x \kappa} = 6\pi\eta R \cdot v$$

(2)



$$p(x) = p_0 \cdot \exp\left(-\frac{U(x)}{k_B T}\right)$$

$$= p_0 \cdot \exp\left(-\frac{\frac{1}{2}\kappa x^2}{k_B T}\right)$$

$$= p_0 \cdot \exp\left(-\frac{\kappa x^2}{2k_B T}\right)$$

$$\propto \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$\boxed{\sigma^2 = \frac{k_B T}{\kappa}} \Rightarrow \boxed{\kappa = \frac{k_B T}{\sigma^2}}$$