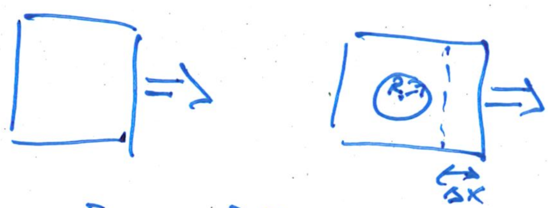
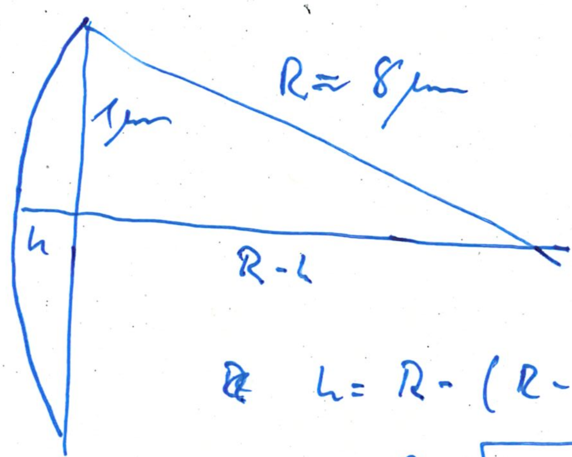
Test



$$\Delta E_+ = \Delta A \cdot \sigma$$

$$\Delta E_L = -\Delta L \cdot \lambda$$

Bending



$$h = R - (R - h)$$

$$= R - \sqrt{R^2 - 1\mu m^2}$$

Estimate Thermal Fluctuation spectrum of a Membrane

$$\Delta E = \iint \left[\frac{1}{2}\sigma(\nabla h)^2 + \frac{1}{2}\kappa_b \cdot (\nabla^2 h)^2\right] dx\,dy$$

q - Modes

$q = \frac{2\pi}{\lambda}$ wavelength

$$h(\vec{x}) = \frac{A}{(2\pi)^2}\int h(\vec{q}) \cdot \exp(i\vec{x}\vec{q})\, d\vec{q}$$

$$\frac{\partial h}{\partial x} = \frac{A}{(2\pi)^2} \cdot \int h(\vec{q}) \cdot iq_x \cdot \exp(i\vec{x}\vec{q})\, d\vec{q}$$

Recall
$$\delta(\vec{q} - \vec{q}') = \frac{1}{(2\pi)^2} \int \exp(ix(\vec{q} - \vec{q}'))\, d\vec{x}$$

Need to square $\frac{\partial h}{\partial x}$

$$\left(\frac{\partial h}{\partial x}\right)^2 = \frac{A^2}{(2\pi)^4} \cdot \iint h(\vec{q})\, h^*(\vec{q}') \cdot (iq_x)(-iq_x') \cdot \exp\left(ix(\vec{q} - \vec{q}')\right) d\vec{q}\, d\vec{q}'$$

$$\int \left(\frac{\partial h}{\partial x}\right)^2 dx\,dy = \frac{A^2}{(2\pi)^2} \int h(q)\, h^*(q) \cdot q_x^2\, d\vec{q}$$

$$\Rightarrow \quad h(q) \cdot h^*(q) = |h(q)|^2$$

$$\int \left(\frac{\partial^2 h}{\partial x^2}\right) d\vec{x} = \frac{A^2}{(2\pi)^2} \int |h(q)|^2 \cdot q_x^4 \cdot d\vec{q}$$

Recall: $\Delta E = \int \left[\frac{1}{2}\sigma (\nabla h)^2 + \frac{1}{2}\kappa_b (\nabla^2 h)^2 \right] d\vec{x}$

$$= \frac{A^2}{(2\pi)^2} \cdot \int \underline{\left[\frac{1}{2} |h(q)|^2 \cdot \left(\sigma \vec{q}^2 + \kappa_b \cdot \vec{q}^4 \right) \right]} d\vec{q}$$

$$\Delta E = \frac{A}{(2\pi)^2} \int E_q \, d\vec{q}$$

$$\Rightarrow \quad E_q = A \cdot \frac{1}{2} |h(q)|^2 \cdot \left(\sigma q^2 + \kappa q^4 \right)$$

Equipartition: $\langle E_q \rangle = \frac{1}{2} k_B T$

$$\cancel{\frac{1}{2}} k_B T = \langle |h(q)|^2 \rangle \cdot \cancel{\frac{1}{2}} A \left(\sigma q^2 + \kappa q^4 \right)$$

$$\Rightarrow \quad \boxed{\langle |h(q)|^2 \rangle = \frac{k_B T}{A \cdot \left(\sigma q^2 + \kappa q^4 \right)}}$$