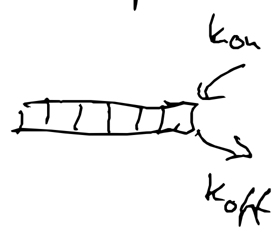
The life of a single filament

Length distribution?



$[P_1] :=$ monomer

$[P_2] :=$ dimer

$\vdots$

$[P_n] =$ n-mer $\quad P_n + P_1 \underset{k_{off}}{\overset{k_{on}}{\rightleftharpoons}} P_{n+1} \iff K_d = \dfrac{[P_n]\cdot[P_1]}{[P_{n+1}]}$

$$\hookrightarrow [P_{n+1}] = \frac{[P_n]\cdot[P_1]}{K_d}$$

$$n=1 \Rightarrow [P_2] = \frac{[P_1][P_1]}{K_d} = \frac{[P_1]^2}{K_d}$$

$$n=2 \Rightarrow [P_3] = \frac{[P_2]\cdot[P_1]}{K_d} = \frac{[P_1]^3}{K_d^2}$$

$$\hookrightarrow \text{General } [P_n] = \frac{[P_1]^n}{K_d^{n-1}} = K_d\left(\frac{[P_1]}{K_d}\right)^n$$

$$= K_d \cdot \exp\Big(n \underbrace{\ln \frac{[P_1]}{K_d}}_{-\alpha}\Big) \qquad \underline{\alpha} = -\ln\frac{[P_1]}{K_d}$$

$$= K_d \cdot \exp(-\alpha n) \qquad p(n) := \frac{\exp(-\alpha n)}{N}$$

$$\int_0^\infty p(n)\,dn = 1$$

What is average length:

$$\langle n\rangle = \int_0^\infty n\cdot p(n)\,dn$$

$$= \frac{\int_0^\infty n\cdot \cancel{K_d}\cdot \exp(-\alpha\cdot n)\,dn}{\int_0^\infty \cancel{K_d}\cdot\exp(-\alpha\cdot n)\,dn}$$

$$f(\alpha) = \int_0^\infty \exp(-\alpha n)\,dn$$

$$\langle n\rangle = \frac{\int_0^\infty n\cdot\exp(-\alpha\cdot n)\,dn}{\int_0^\infty \exp(-\alpha\cdot n)\,dn}$$

$$\frac{d}{d\alpha}\ln(f(\alpha)) = \frac{1}{f(\alpha)}\cdot f'(\alpha)$$

$$= \frac{-\frac{\partial}{\partial\alpha}\int_0^\infty \exp(-\alpha\cdot n)\,dn}{\int_0^\infty \exp(-\alpha n)\,dn}$$

$$= -\frac{\partial}{\partial\alpha}\left[\ln\left(\int_0^\infty \exp(-\alpha n)\,dn\right)\right]$$

$$= -\frac{\partial}{\partial\alpha}\ln\left(\frac{1}{\alpha}\right) \Rightarrow \langle n\rangle = \frac{1}{\alpha}$$

$$\alpha = -\ln\frac{[P_1]}{K_d}$$